Commutativity (Joy, Ch 4) & Percentages

Multiplying Whole Numbers

7×3 "seven threes" 3×7 " three   sevens"
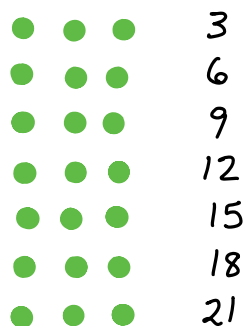
This is the commutative law for numbers. If a and b
are numbers, then _____.

More Subtle : Discount & Sales Tax

<u>Ex</u> $50 jeans, 20% off, 8% tax

Recall some basics about percents

- Percent : _____

    20% means [   ]

- Used to describe _____

    percent = part/whole &   part = percent × whole

<u>Ex</u>   20% of $50 :

[   ]

<u>Ex</u>   3 out of 12 in our class

[   ]

- Note : May have percent higher than 100% ($^{11}/_{10}$ is 110%)

Jeans Example : Additive Approach

$50 jeans, 20% off, 8% sales tax

(1) Discount on orig. price is 20% of $50.

$$(0.2) \times (\$50) = \boxed{}$$

So sale price is:

$$\$50 - \boxed{} = \$\boxed{}$$

Tax on sale price: 8% of $40

$$\boxed{} \times (\$40) = \boxed{}$$

So final price is:

$$\$40 + \boxed{} = \boxed{}$$

(2) Tax on orig price: 8% of $50

$$(0.08) \times (\$50) = \boxed{}$$

Orig. price + tax:

$$\$50 + \boxed{} = \boxed{}$$

Discount on $54:

$$(0,2) \times (\$54) = \boxed{}$$

Final price:

$$\$54 - \boxed{} = \boxed{}$$

+ tax

$50

− discount

− discount

+ tax
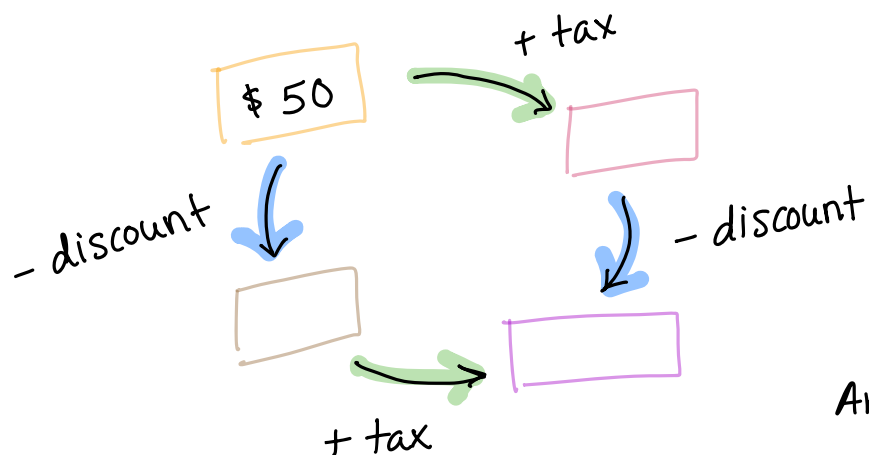
Amazing!

Multiplicative Approach

Apply 20% discount to some amount:

$$(amt.) - (0.2) \times (amt)$$

$$= \boxed{} = \boxed{} \times (amt) \leftarrow \boxed{} \text{ of amt.}$$

Apply 8% tax to some amount:

$$(amt.) + (0.08) \times (amt)$$

$$= \boxed{} = \boxed{} \times (amt) \leftarrow \boxed{} \text{ of amt.}$$

$$(\$50) \times (0.8) \times (1.08)$$
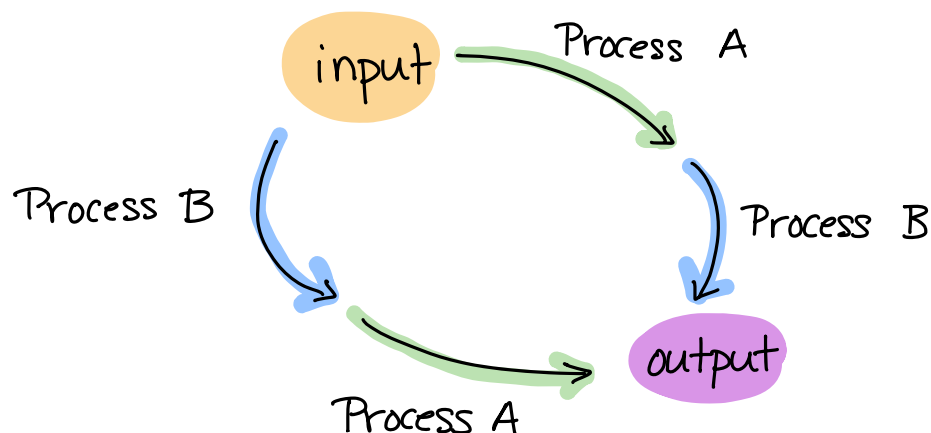
$$(\$50) \times (1.08) \times (0.8)$$

$$\boxed{}$$
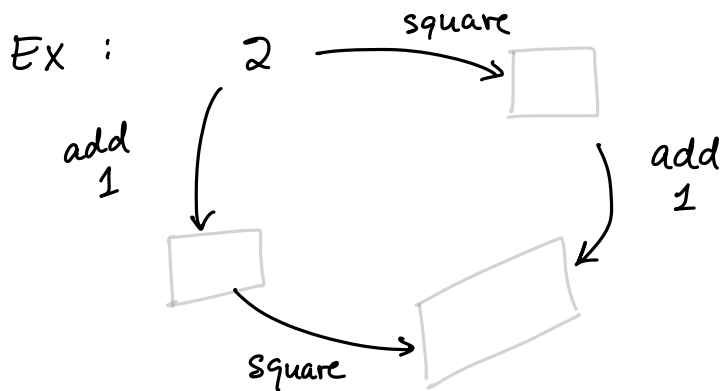
Commutativity More Generally

Two processes, "Process A" and "Process B," are said to _____ if :

Given any specific input, we may perform the processes in _____ and get the _____.

- socks & shoes

- brush & floss?

- square & add 1

  Ex :   2 $\xrightarrow{\text{square}}$ □

- multiplication of "matrices"

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} =$$

$$\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} =$$

Quantum Mechanics $p \times q \neq q \times p$.

Percents in Daily Life

More About Percents

Absolute & Relative Change

__Ex__ Cost of item went up by $2 vs by 11%

__Ex__ Two churches each grow by 50 members

    (1) __Initial__ membership 200:

        rel. change: $50/\boxed{} = \boxed{}$

    (2) Initial membership 1000:

        rel. change: $50/\boxed{} = \boxed{}$

Base of Percent

Tells what you are comparing to.

__Ex__ 80% of marijuana users end up on hard drugs (F)

   vs. 80% of hard drug users started w/ marijuana (T)

Impactful for public policy!

80% of marijuana users end up on hard drugs

   • base is # of marijuana users

$$0.8 = \left(\begin{array}{c}\text{\# m. users who ended} \\ \text{up on hard drugs}\end{array}\right) / \left(\begin{array}{c}\text{total \# of} \\ \text{m. users}\end{array}\right)$$

80% of hard drug users started w/ marijuana

   • base is # of hard drug users

$$0.8 = \left(\begin{array}{c}\text{\# hard drug users who} \\ \text{started w/ marijuana}\end{array}\right) / \left(\begin{array}{c}\text{total \# of hard} \\ \text{drug users}\end{array}\right)$$

                 ↑

        same as prev.

Ex   % of high school drop-outs, who failed reading in 3rd grade
- base is # _____

vs.   % of people who failed reading in 3rd grade, who ended up dropping out of high school
- base is # _____

Change in Percent

Use phrase "percentage points" for absolute change in a percent

Ex   Ave test score for Calc 3   Exam 1 :   83%

       "   "   "   "   "   Exam 2 :   81%

Ave. test score decreased by 2 _____.

Next Class :

Work as a group to solve problems involving percents in your lives.

To prepare :
- Write 3 "story" problems : percents in your life
- Include :   % increase, % decrease
                        absolute & relative change

- Bring several copies
  - one for each member of your group
  - one to turn in (or turn in online)

- Bring calculator.

Quiz 1 next Fri.   See Canvas for details.